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Yukawa Couplings for the Spinning Particle and the Worldline Formalism

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Abstract

We construct the world-line action for a Dirac particle coupled to a classical scalar or pseudo-scalar background field. This action can be used to compute loop diagrams and the effective action in the Yukawa model using the world-line path-integral formalism for spinning particles.

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In the framework of the Bern-Kosower formalism [1, 2, 3], derived from string theory, it is possible to reproduce results of tree and one-loop field theory calculations in a compact and elegant form which has its origin in the simplicity of the string theory perturbation expansion. Many of the results for one-loop calculations can also be understood through world-line path integrals [4, 5]. This approach to quantum field theory can be generalised to multi-loop calculations [6, 7]. For earlier applications of world-line path integrals to quantum field theory see also [8, 9, 10, 11].

In the following we construct the action for the Yukawa coupling for the spinning particle in Euclidean space. While the original Bern-Kosower approach works for Yukawa couplings [12] the correct form of the world-line action was so far unknown, even though guesses existed before [4]. Our starting point is the well-known world-line action for a massless, spinning particle coupled to a gauge field [13, 14]. By dimensional reduction of the gauge coupling we obtain the form of the Yukawa coupling. For the case of a constant background field we recover the action for the massive, spinning particle as a special case. It turns out that we can generalise this to include pseudo-scalar couplings as well as scalar couplings.

We present some applications to the calculation of one-loop amplitude both to reproduce the result of Feynman diagram calculations and to get a variant of the heat-kernel expansion for the one-loop effective action.

Our starting point is the first-quantized description of a Dirac particle [15, 13, 14], given by a supersymmetric action in one dimension. Such an action can be formulated in a compact way using superfield notation. Furthermore, this notation ensures the supersymmetry of the action, an important point since we want to add new couplings without loosing supersymmetry.

Our interest lies in the application of the world-line formalism to loop calculations with external scalar fields. For this reason we do not have to worry about the problem of boundary conditions for the fermions like in the derivation of the Dirac propagator from the world-line action [16, 17].

In the superfield formulation [15], the world-line parameter τ is supplemented by an anti-commuting Grassmann parameter θ to form a two-dimensional superspace. In this formulation, the free spinning particle is described using world-line superfields with a space-time vector index

$$X^\mu(\tau,\theta) = x^\mu(\tau) + \theta\sqrt{e}\,\psi^\mu(\tau), \tag{1}$$

where x is a normal commuting number, and θ is a Grassmann variable.

To be able to write down a reparametrisation invariant action we need to introduce the super-einbein [15]

$$\Lambda = e + \theta\sqrt{e}\,\chi. \tag{2}$$

In curved superspace, two independent derivatives exist:

$$\begin{aligned} D_\theta &= \Lambda^{-1/2}\left(\frac{\partial}{\partial\theta} - \theta\frac{\partial}{\partial\tau}\right), \\ D_\tau &= \Lambda^{-1}\frac{\partial}{\partial\tau}. \end{aligned} \tag{3}$$

Using these ingredients, the world-line action for a free, spinning particle is

$$S_0 = \frac{1}{2}\int \mathrm{d}\tau\mathrm{d}\theta\,\Lambda^{1/2} D_\tau X\cdot D_\theta X. \tag{4}$$

In components, this reads

$$S_0 = \frac{1}{2}\int \mathrm{d}\tau \left(\frac{\dot{x}^2}{e} + \frac{1}{e}\chi\dot{x}\psi + \psi\dot{\psi} \right). \tag{5}$$

The coupling of the spinning particle to a Yang-Mills field is well-known [13]:

$$S_{\mathrm{YM}} = \int \mathrm{d}\tau \mathrm{d}\theta\, \Lambda^{1/2} i g D_\theta X_\mu A^\mu. \tag{6}$$

If we take this coupling in five dimensions and analyze it from a four-dimensional point of view, we find a Dirac-spinor with both Yang-Mills and Yukawa couplings. By choosing a background field such that $A_\mu = 0$ for $\mu = 0, \ldots, 3$ we can get a system which has only one Yukawa coupling.

In the process of dimensional reduction we single out the fields X_5 and A_5. Furthermore, we notice that X_5 only appears in the combination DX_5. Therefore we introduce the following convenient definitions:

$$\bar{X} \equiv \Lambda^{1/2} D_\theta X_5, \qquad \Phi \equiv A^5. \tag{7}$$

With these definitions the world-line action for a spinning particle with a Yukawa coupling is

$$S_{\mathrm{Y}} = S_0 + \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}\theta\, \Lambda^{1/2} \left(\Lambda^{-1} \bar{X} D_\theta \bar{X} + 2\Lambda^{-1/2} i\lambda \bar{X} \Phi(X) \right). \tag{8}$$

To rewrite this expression in components we expand the superfield $\bar{X}$, which is fermionic, and the scalar field Φ as

$$\bar{X} = \sqrt{e}\,\psi_5 + \theta x_5 \qquad \text{and} \qquad \Phi(X) = \Phi(x) + \theta\sqrt{e}\,\psi^\mu \partial_\mu \Phi(x). \tag{9}$$

(This expansion defines x_5 for the rest of this letter, not eq. (1). We apologise for the confusion.) In component notation, the action (8) is

$$\begin{aligned} S_{\mathrm{Y}} = \frac{1}{2}\int \mathrm{d}\tau \Bigg\{ & \frac{\dot{x}^2}{e} + \frac{x_5^2}{e} + \psi\cdot\dot{\psi} + \psi_5\dot{\psi}_5 + \chi\left(\frac{1}{e}\dot{x}\psi - x_5\psi_5\right) \\ & + 2i\lambda\Big(x_5\Phi(x) - e\psi_5\psi\cdot\partial\Phi(x)\Big) \Bigg\}. \end{aligned} \tag{10}$$

To introduce a mass term for the fermions, all we have to do is to add a constant piece to the scalar field. If we shift the scalar field Φ by a constant $\Phi \to \Phi + m/\lambda$ we introduce a mass term into our action. This procedure introduces a term of the form $2ix_5 m$ which turns out to be an inconvenience when we want to construct the perturbation expansion (eq. (15)). To eliminate this term, we shift $x_5 \to x_5 - iem$ and get

$$\begin{aligned} S_{\mathrm{Y}} = \frac{1}{2}\int \mathrm{d}\tau \Bigg\{ & \frac{\dot{x}^2}{e} + \frac{x_5^2}{e} + em^2 + \psi\cdot\dot{\psi} + \psi_5\dot{\psi}_5 + \chi\left(\frac{1}{e}\dot{x}\psi - (x_5 - iem)\psi_5\right) \\ & + 2\lambda\Big((ix_5 + em)\Phi(x) - ie\psi_5\psi\cdot\partial\Phi(x)\Big) \Bigg\}. \end{aligned} \tag{11}$$

From this we can integrate out the auxiliary field x_5,*i. e.,* we eliminate it using its equation of motion:[1]

$$\begin{aligned} S_{\mathrm{Y}} = \frac{1}{2}\int \mathrm{d}\tau \Bigg\{ & \frac{\dot{x}^2}{e} + \psi\cdot\dot{\psi} + \psi_5\dot{\psi}_5 + em^2 + 2e\lambda m\Phi + e\lambda^2\Phi^2 \\ & - 2i\lambda e\psi_5\psi\cdot\partial\Phi(x) + \chi\left(\frac{1}{e}\dot{x}\psi + iem\psi_5 + ie\lambda\psi_5\Phi\right) \Bigg\}. \end{aligned} \tag{12}$$

[1] This action differs from the action proposed in [4]. Starting from this suggestion, we were not able to perform the calculations analogous to our one-loop examples presented here.

If we set $\Phi = 0$, we recover the action for the *massive*, spinning particle in component notation [15, 13] or, from eq. (8), the action in superfield language [18].

So far, we discussed the scalar coupling $i\lambda\Phi\bar{\psi}\psi$. It is also possible to find a world-line action which reproduces the one-loop effective action for the Yukawa model with the pseudo-scalar coupling $\lambda'\Phi'\bar{\psi}\gamma_5\psi$. The only change necessary is the introduction of another fermionic superfield $X' = x_6 + \theta\sqrt{e}\,\psi_6$ for the new interaction term. The presence of the two fields $\bar{X}$ and X' ensures that the terms generated from the scalar and pseudo-scalar interactions do not mix — something that the γ_5-matrix ensures in the standard field theory treatment.

For a massive, scalar field with a pseudo-scalar coupling the resulting action is then

$$S_{\mathrm{PS}} = \frac{1}{2}\int \mathrm{d}\tau\mathrm{d}\theta\,\Lambda^{1/2}\Big(D_\tau X\cdot D_\theta X + \Lambda^{-1}\bar{X}D_\theta\bar{X} + \Lambda^{-1}X'D_\theta X' + \Lambda^{1/2}im\bar{X} + \Lambda^{1/2}i\lambda' X'\Phi'(X)\Big)\,. \tag{13}$$

In components (in the $\chi = 0$ gauge), after the elimination of the auxiliary fields, this reads

$$S_{\mathrm{PS}} = \frac{1}{2}\int \mathrm{d}\tau\left\{\frac{\dot{x}^2}{e} + \psi\cdot\dot{\psi} + \psi_6\dot{\psi}_6 + em^2 + e\lambda'^2\Phi'^2 - 2i\lambda' e\psi_6\psi\cdot\partial\Phi'(x)\right\}. \tag{14}$$

We could also formulate this action analogous to eq. (11) or, from eqs. (12) and (14), write down the action for a spinning particle which has both scalar and pseudo-scalar Yukawa couplings.

Now that we have a world-line action for a spinning particle with a Yukawa coupling we want to apply it to some one-loop calculations. The starting point is always the world-line expression for the one-loop effective action [4, 5]

$$\Gamma(\Phi) = -2\int_0^\infty \frac{\mathrm{d}T}{T}\int \mathcal{D}x\mathcal{D}x_5\mathcal{D}\psi\mathcal{D}\psi_5\, e^{-TS_{\mathrm{Y}}}. \tag{15}$$

This can be used both for deriving rules for the calculation of one-loop n-point functions and as an expression from which one can derive approximations to the one-loop effective action itself.

For any calculation we need rules how to evaluate the path integral in eq. (15). In principle, the path integral also includes the integration over the fields e and χ. Since infinitesimal changes in those fields are associated with infinitesimal reparametrisations we have a gauge invariance in our system. After treating this with standard methods [19] we are left with the conventional integral over T, where T labels inequivalent circles. This gauge-fixing is the origin of the factor $\mathrm{d}T/T$ in eq. (15). The free $\mathcal{D}x$ path-integral is normalised to $(4\pi T)^{-d/2}$, the other free path integrals are 1 [19, 6]. A common [13, 4, 5], convenient gauge choice is $e = 2$ and $\chi = 0$.

Before proceeding, we separate the centre of mass x_0 from the embedding coordinate x as

$$x(\tau) \equiv x_0 + y(\tau), \qquad \text{with} \qquad \int_0^T \mathrm{d}\tau\, y(\tau) = 0. \tag{16}$$

The remaining path integral can be evaluated using standard methods. From the free part we obtain correlation functions for the quantum fields. And the exponential containing the interaction gets expanded according to the chosen approximation. The interaction part is

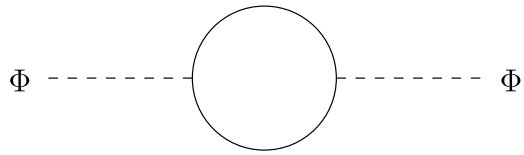

Figure 1: The two-point function $\Gamma_\Phi^{(2)}$ for the scalar field.

evaluated using Wick contractions with the correlation functions [4, 5]

$$
\begin{aligned}
\langle y^\mu(\tau_1) y^\nu(\tau_2)\rangle &= -g^{\mu\nu} G_B(\tau_1,\tau_2), \qquad G_B(\tau_1,\tau_2) = |\tau_1-\tau_2| - \frac{(\tau_1-\tau_2)^2}{T}, \\
\langle \psi^\mu(\tau_1)\psi^\nu(\tau_2)\rangle &= \frac{1}{2} g^{\mu\nu} G_F(\tau_1,\tau_2), \qquad G_F(\tau_1,\tau_2) = \mathrm{sign}(\tau_1-\tau_2), \\
\langle \psi_5(\tau_1)\psi_5(\tau_2)\rangle &= \frac{1}{2} G_F(\tau_1,\tau_2),
\end{aligned} \tag{17}
$$

and

$$
\langle x_5(\tau_1) x_5(\tau_2)\rangle = 2\delta(\tau_1,\tau_2). \tag{18}
$$

The Green's functions are the ones on the circle for bosonic fields with periodic boundary conditions, and for fermions with anti-periodic boundary conditions. The extra term in the G_B results from the background charge required due to the compact nature of the circle. To find the Green's function (18) for x_5, one just has to invert the identity operator.

As a simple example indicating how to reproduce the results of a standard Feynman diagram calculation, let us look at the two-point function for the scalar field in the Yukawa model (fig. 1).

From the standard expression

$$
(2\pi)^d \delta(p_1+p_2)\Gamma_\Phi^{(2)}(p_1,p_2) = \frac{\delta^2}{\delta\Phi(p_1)\delta\Phi(p_2)}\Gamma(\Phi)\Big|_{\Phi=0} \tag{19}
$$

we get the world-line expression for the spinor loop correction to the scalar two-point function. From the Fourier representation

$$
\Phi(x) = \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{-ip\cdot x}\Phi(p) \tag{20}
$$

the functional differentiation in eq. (19) automatically generates the vertex operator $\Phi \to \exp(-ip\cdot x)$ for the scalar field. The expression for the scalar propagator correction is then

$$
\begin{aligned}
(2\pi)^d \delta(p_1+p_2)\Gamma_\Phi^{(2)}(p_1,p_2) &= -2\int_0^\infty \frac{\mathrm{d}T}{T}(4\pi T)^{-d/2} e^{-Tm^2}\int \mathrm{d}^d x_0\, e^{-ix_0\cdot(p_1+p_2)} \times \\
&\Bigg[-2\int_0^T \mathrm{d}\tau\, \lambda^2 \langle e^{-ip_1\cdot y(\tau)} e^{-ip_2\cdot y(\tau)}\rangle \\
&+ \int_0^T \mathrm{d}\tau_1 \int_0^T \mathrm{d}\tau_2 \Big\{ 4\lambda^2 m^2 \langle e^{-ip_1\cdot y(\tau_1)} e^{-ip_2\cdot y(\tau_2)}\rangle \\
&- 4\lambda^2 \langle\psi_5(\tau_1)\psi_5(\tau_2)\rangle\langle\psi^\mu(\tau_1)\psi^\nu(\tau_2)\rangle p_{1,\mu}p_{2,\nu}\langle e^{-ip_1\cdot y(\tau_1)} e^{-ip_2\cdot y(\tau_2)}\rangle\Big\}\Bigg].
\end{aligned} \tag{21}
$$

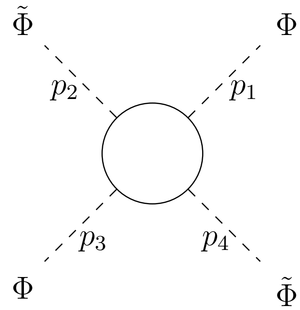

Figure 2: The mixed four-point function.

Besides the correlation functions from eq. (17) we use the contraction of exponentials

$$\langle e^{-ip_1\cdot x(\tau_1)} e^{-ip_2\cdot x(\tau_2)}\rangle = e^{p_1\cdot p_2 G_B(\tau_1,\tau_2)} \tag{22}$$

to evaluate (21).

The x_0-integral produces the momentum conserving δ-function which we use to set $p \equiv p_1 = -p_2$. To evaluate this expression further we use the standard rescaling $\tau_i = Tu_i$ [4, 5, 20]. This way, all the T-dependence is displayed explicitly and the T-integration can be done, leading to

$$\Gamma^{(2)}_{\Phi}(p,-p) = -2\lambda^2(4\pi)^{\epsilon-2}\Big(-2\Gamma(\epsilon-1)(m^2)^{1-\epsilon} + \Gamma(\epsilon)\left[4m^2+p^2\right]\int_0^1 \mathrm{d}x\,\left(m^2+p^2x(1-x)\right)^{-\epsilon}\Big) \tag{23}$$

where we introduce $\epsilon = 2 - d/2$.

The result of a standard Feynman parameter calculation is

$$\Gamma^{(2)}_{\Phi}(p,-p) = -2\lambda^2(4\pi)^{\epsilon-2}\int_0^1 \mathrm{d}x\,\left(\left[(2+\epsilon)\Gamma(\epsilon-1)+\Gamma(\epsilon)\right](m^2+p^2x(1-x))^{1-\epsilon}\right) \tag{24}$$

from which we can recover result (23) by judicious integration by parts.

It is worthwhile to note here that we do not have the straightforward connection between the τ-intervals of the world-line formalism and the Feynman α-parameters. Alternatively, starting from a second-order expression for the fermion one-loop effective action [3], it is possible to reorganise the field theory perturbation series in a manner analogous to the world-line formalism[2] (alas, only after performing the integration over the loop momentum).

The difference in the organisation of the perturbation series will be even more noticeable in the next example where it is not possible to isolate the contribution of a single (first-order formalism) Feynman diagram in the expression generated by the world-line formalism.

To illustrate the point we just made, let us look at the four-point function (fig. 2) where, in a standard field theory calculation, six Feynman diagrams contribute. The world-line formalism generates only one expression which cannot be divided by restricting the region of integration of the τ's into the contribution corresponding to individual Feynman diagrams.

[2]We would like to thank D. C. Dunbar for discussions on that point.

From world-line action (14) we immediately get the expression

$$
\begin{aligned}
\Gamma^{(4)}(p_1,p_2,p_3,p_4) = -2\int_0^\infty \frac{\mathrm{d}T}{T}(4\pi T)^{-d/2}e^{-Tm^2} \times \\
\times \Bigg[4\lambda^2\lambda'^2 \int_0^T \mathrm{d}\tau_1 \int_0^T \mathrm{d}\tau_2 \left\langle e^{-ip_1\cdot y_1}e^{-ip_3\cdot y_1}e^{-ip_2\cdot y_2}e^{-ip_4\cdot y_2}\right\rangle \\
- 8\lambda^2\lambda'^2 \int_0^T \mathrm{d}\tau_1 \int_0^T \mathrm{d}\tau_2 \int_0^T \mathrm{d}\tau_3 \times \\
\Big\{ (m^2 - \langle\psi_{5,1}\psi_{5,2}\rangle\langle\psi_{\mu,1}\psi_{\nu,2}\rangle\, p_1^\mu p_3^\nu) \left\langle e^{-ip_1\cdot y_1}e^{-ip_3\cdot y_2}e^{-ip_2\cdot y_3}e^{-ip_4\cdot y_3}\right\rangle \\
- \langle\psi_{6,1}\psi_{6,2}\rangle\langle\psi_{\mu,1}\psi_{\nu,2}\rangle\, p_2^\mu p_4^\nu \left\langle e^{-ip_1\cdot y_3}e^{-ip_3\cdot y_3}e^{-ip_2\cdot y_1}e^{-ip_4\cdot y_2}\right\rangle \Big\} \\
+ 16\lambda^2\lambda'^2 \int_0^T \mathrm{d}\tau_1 \int_0^T \mathrm{d}\tau_2 \int_0^T \mathrm{d}\tau_3 \int_0^T \mathrm{d}\tau_4 \times \\
\Big\{ -m^2\langle\psi_{6,2}\psi_{6,4}\rangle\langle\psi_{\mu,2}\psi_{\nu,4}\rangle\, p_2^\mu p_4^\nu \qquad (25) \\
+ \langle\psi_{5,1}\psi_{5,3}\rangle\langle\psi_{6,2}\psi_{6,4}\rangle\langle\psi_{\mu,1}\psi_{\nu,2}\psi_{\rho,3}\psi_{\sigma,4}\rangle\, p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \Big\} \times \\
\left\langle e^{-ip_1\cdot y_2}e^{-ip_2\cdot y_2}e^{-ip_3\cdot y_3}e^{-ip_4\cdot y_4}\right\rangle \Bigg] .
\end{aligned}
$$

Here we use the short-hand notation $y_i \equiv y(\tau_i)$, and we omitted the momentum conserving δ-function. The scalar fields are attached to the momenta p_1 and p_3 while the pseudo-scalar fields are on p_2 and p_4.

With the contractions (17), it is easy to evaluate this expression. The T-integrations lead again to the Γ-functions of dimensional regularisation, and the G_B's produce polynomials similar to polynomials in Feynman parameters in the standard field theory calculation. Whereas the T-integration is simple, the remaining τ-integrations are more complicated — they are of a form similar to scalar bubble, triangle and box integrals. To illustrate our point it suffices to extract the $1/\epsilon$-term from eq. (25). The only divergence sits in the first term which is

$$
-\frac{8\lambda^2\lambda'^2}{(4\pi)^{2-\epsilon}}\Gamma(\epsilon)\int_0^1 \mathrm{d}u \,\Big(m^2 + (p_1\cdot p_2 + p_1\cdot p_4 + p_3\cdot p_2 + p_3\cdot p_4)u(1-u)\Big)^{-\epsilon} . \qquad (26)
$$

This diverges for $\epsilon \to 0$ as $-\lambda^2\lambda'^2/(2\pi^2\epsilon)$. The same result is easily obtained from the usual evaluation of Feynman diagrams in the first-order formalism. In that case, however, the contributions of different diagrams enter with different signs. Such cancelations do not occur in the world-line formalism — we immediately get the answer for the sum of several Feynman diagrams. The point we made before should be clear after this example: In general, in the world-line formalism we compute the sum of a class of Feynman diagrams. It is not always possible to identify the contribution of a single Feynman diagram by restricting the integration over the world-line parameters τ_i.

From the effective action (15) we can also generate very elegantly the asymptotic heat-kernel expansion of the effective action [21, 22, 23, 24].

Instead of working in momentum space, as we did in the previous section, we work now in coordinate space. The field Φ and its derivative are now expressed through their Taylor expansion as [5]

$$
\Phi(x(\tau)) = e^{y(\tau)\cdot\partial}\Phi(x_0) \qquad (27)
$$

about the centre of mass x_0. With this we can proceed as before and expand the interaction part of the exponential in (15), perform the contractions, and rescale the integration variables. However, instead of performing the T-integration we expand the exponentials of the form $\exp(-TG_B(u_i,u_j)\partial_i\cdot\partial_j)$, generated by (22). After arranging the series by powers of T and performing the u_i integrals one arrives at a variant [5, 25] of the asymptotic heat-kernel expansion, which is particularly well organised [26]. In a forthcoming publication [27] we will discuss in detail this expansion in the case of the Yukawa model. The calculation of the expansion of the effective action with different methods gives us another check that eq. (15) indeed is equivalent to the usual effective action in field theory. We verified this for the scalar and pseudo-scalar couplings separately as well as for the mixed case.

We managed to construct the world-line action for the Yukawa model both for scalar and pseudo-scalar couplings. For some simple examples we showed how we can reproduce the result of a standard Feynman diagram calculation using our action in the world-line formalism. We performed more calculations to verify the agreement in other cases which, for the sake of brevity, we do not present in this letter.

It has been noted in [6] and further exemplified in [28] that the world-line path integral formalism, if used with a global parametrisation, naturally offers the possibility of combining diagrams of different topology into a single master expression. (For a recent application of this idea, see also [29]). It will be interesting to see whether this leads to simplifications in the case of the two-loop box diagrams in the Yukawa model, which can be treated with the methods developed in this paper.

Acknowledgments: We would like to acknowledge the help of D. Fliegner and discussions with P. Haberl.

Note added: Already in the massive Yukawa model with only a pseudoscalar coupling, amplitudes with an odd number of vertices are possible and give rise to ϵ-tensor terms. We thank D. Geiser for pointing this out to us. These terms will be dealt with in a forthcoming publication treating axial couplings.

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